in either case. With the help of (1), we are led to

$$\begin{aligned}\left(\sum_{k=1}^{n+1}\cosh x_k\right) &\geq \cosh x_{n+1} + n - 1 + \cosh\left(\sum_{k=1}^{n}(-1)^{k-1}x_k\right)\\ &\geq n + \cosh\left(\sum_{k=1}^{n+1}(-1)^{k-1}x_k\right).\end{aligned}$$

This completes the induction step and the proof.

4095. *Proposed by George Apostolopoulos.*

Let a, b and c be positive real numbers with $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=3$. Prove that

$$ab(a+b)+bc(b+c)+ac(a+c) \geq \frac{2}{3}(a^2+b^2+c^2)+4abc.$$

There were 14 correct solutions. We present six different ones here.

Solution 1, by Arkady Alt.

With $1 = ax = by = cz$ and $x+y+z=3$, the inequality is equivalent to

$$\begin{aligned}&(x+y+z)[z^2(x+y)+x^2(y+z)+y^2(z+x)]\\ &\geq 2(x^2y^2+y^2z^2+z^2x^2)+4(x+y+z)(xyz)\\ &= 2(xy+yz+zx)^2.\end{aligned}$$

The difference between the two sides is

$$\begin{aligned}&(x+y+z)[(x+y+z)(xy+yz+zx)-3xyz]-2(xy+yz+zx)^2\\ &= (x+y+z)^2(xy+yz+zx)-2(xy+yz+zx)^2-3xyz(x+y+z)\\ &\geq 3(xy+yz+zx)(xy+yz+zx)-2(xy+yz+zx)^2-3xyz(x+y+z)\\ &= (xy+yz+zx)^2-3xyz(x+y+z)\geq 0\end{aligned}$$

(from two applications of the inequality $(u+v+w)^2 \geq 3(uv+vw+wu)$). Equality occurs iff $1=x=y=z=a=b=c$.

Solution 2, by Andrew Siefker and Digby Smith (done independently).

When a, b, c, x, y, z are all positive, we have

$$\begin{aligned}&(a^2yz+b^2zx+c^2xy)(x+y+z)-(a+b+c)^2xyz\\ &= z[(a^2y^2+b^2x^2-2abxy]+y[a^2z^2+c^2x^2-2acxz]+x[b^2z^2+c^2y^2-2bcyz]\geq 0,\end{aligned}$$

so that

$$\frac{a^2}{x}+\frac{b^2}{y}+\frac{c^2}{z}\geq\frac{(a+b+c)^2}{x+y+z}.$$

Equality occurs iff $a:b:c = x:y:z$. (This also results from Cauchy's Inequality.)